Approximating integrals using least-squares best-fitting polynomials

1. A noisy sensor is reading speed at a rate of once every five seconds, and the reading is in meters per second. The readings are as follows:

0, 0, 0, 0, –0.35, 1.84, 1.56, –1.12, –4.70, 2.95, 3.77, 1.97, 5.81, 8.11, 10.62, 11.88, 17.45

Use the five-point approximation shown in the course slides:

1. For best-fitting least-squares linear polynomials:
   $a_1 = -0.2y_{n-4} - 0.1y_{n-3} + 0.1y_{n-1} + 0.2y_n$
   $a_0 = -0.2y_{n-4} + 0.2y_{n-2} + 0.4y_{n-1} + 0.6y_n$
2. For best-fitting least-squares linear polynomials:
   $a_2 = (\ 2y_{n-4} - \ y_{n-3} - \ 2y_{n-2} - \ y_{n-1} + \ 2y_n)/14$
   $a_1 = (26y_{n-4} - 27y_{n-3} - 40y_{n-2} - 13y_{n-1} + 54y_n)/70$
   $a_0 = (\ 3y_{n-4} - \ 5y_{n-3} - \ 3y_{n-2} + \ 9y_{n-1} + 31y_n)/35$

Answer: Starting with the 5^{th} point, assuming all previous integrals are zero, and rounding to two decimal places for the quadratic polynomial:

–0.875, 3.1125, 9.8825, 11.565, 0.05, –2.45, 6.3775, 20.235, 45.065, 76.945, 122.105, 178.9675, 253.125

–0.96, 3.51, 10.51, 11.35, –1.83, –2.55, 8.45, 21.92, 45.53, 78.44, 124.44, 180.71, 255.50

2. The data in Question1 comes from Question 3 in Question Set 5.2.3 with significant noise introduced into the numbers, with a normally distributed value with a standard deviation of two being added to each entry. Compare the integrals.

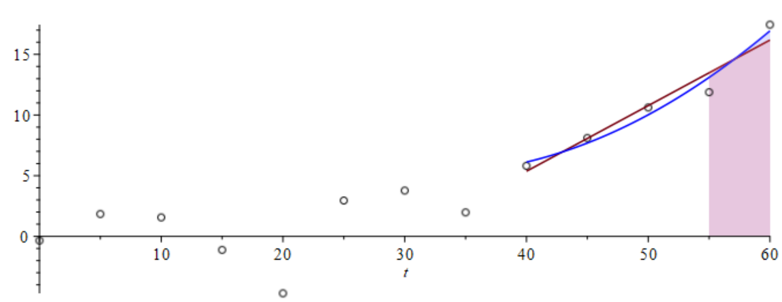
Answer: With such significant noise being added to the data the fact that both solutions have perhaps one-and-a-half significant digits is potentially useful.

3. With as much noise as was introduced into the data in Question 1, would it make more sense, or less sense, to use more points in finding the best-fitting least-squares polynomials?

Answer: The errors introduced into the data is quite significant, so more points would definitely give a much better approximation by eliminating some of that error.

4. Plot the points with noise, and then plot the least-squares best-fitting polynomials that are used to estimate the derivatives at the last point assuming the first noisy signal was taken at time $t = 0$.

Answer:



5. Estimate the integral over the next time interval beyond the next point.

Answer: Using the linear polynomial, $a_1 = 2.705$ and $a_0 = 16.184$, and so our approximation of the integral over the next time step (the distance we will travel in the next five seconds) is

$$(16.184 + 2.705/2)5 = 87.6825$$

Using the quadratic polynomial, to four digits after the decimal point, $a_2 := 0.3779$, $a_1 = 4.2164$, and $a_0 = 16.9397$, and so our approximation of the integral over the next time step is

$$(16.9397 + 4.2164/2 + 0.3779/3)5 = 95.8693$$